“Globally Optimal Stratified Autocalibration”

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Paper Summary

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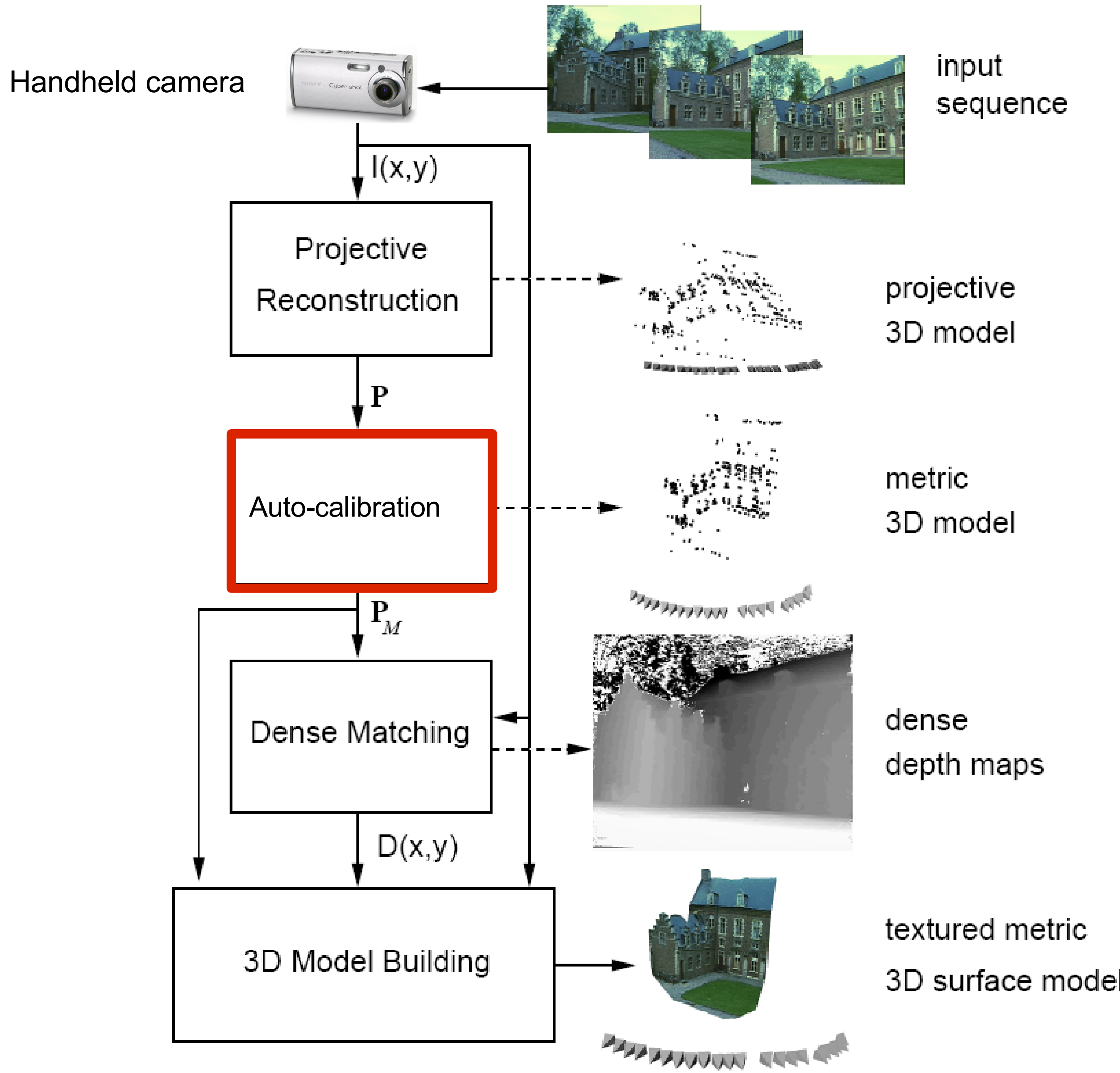
Científico e Tecnológico

Brown

Context


Handheld camera
input
sequence
I(x,y)
Projective
Reconstruction
projective
3D model
P
Auto-calibration
metric
3D model
P_M
Dense Matching
dense
depth maps
D(x,y)
3D Model Building
textured metric
3D surface model

Blackbox description of proposed system

- Input
  - Multiple images
  - Projective reconstruction
  - Rough bounds for the intrinsics
    - Principal point
    - Focal length
    - Aspect ratio
    - Skew
- Output
  - Metric reconstruction
    - Intrinsic parameters, Rotations, and translations up to global scale

Outline of contributions

- New numerical method for solving auto-calibration equations
  - Buzz words:
    - Branch-and-bound deterministic global optimization
    - Convex/concave relaxations
    - “Semi-definite” cone programming
  - Assumes constant intrinsic parameters
- Guaranteed global minimum is reached
  - Algebraic cost with constraints on the solution
  - Classic methods are precarious

Bottom Line

Pros

- Applies more appropriate numerical methods
- Solves computational difficulties in the literature
- Potentially useful methods

Cons

- No experimental comparison with previous methods
- Not novel
  - Same strategy used for autocalibration in
    - Fusiello, Benedetti, Farenzena, Busti PAMI 2004 – Kruppa Eqs
  - Similar CVPR'07 paper by same authors
    - Zero-skew, unit aspect ratio, principal point known. Direct method.
- No significant new insights
  - Authors were sensible in analyzing purely algebraic properties of the usual equations and exploiting this in providing a good algorithm
  - All equations, constraints, and major insights were already available, although with poor numerical methods
- Algebraic cost - authors *still* recommend bundle adjustment in the end

Outline

- Projective reconstruction is given
  - Computing epipolar geometry for all views
- Classic result: remaining DOF of cameras is a 3D projective transformation *H*

$$x_i = P_i X_i = (P_i H)(H^{-1} X_i)$$

$$P_i H^{-1} \text{ and } P_i \text{ give the same epipolar geometry}$$

- This ambiguity has 8DOF
  - 5 for calibration matrix K
    - Synonym: Image of the Absolute Conic (IAC)
  - 3 for non-linear distortion related DOF of rotation and translation of the system
    - Synonym: Plane at Infinity

Auto Calibration Equations in 1 Slide

- Basic idea
  - Assume fully calibrated system
  - All cameras must be consistent with first one
    - Based on epipolar geometry
    - No concrete geometric interpretation
  - This gives equations relating the intrinsic parameters

$$K^i K^{i\top} = (A^i - a^i p^\top) K^1 K^{1\top} (A^i - a^i p^\top)^\top$$

  - ***A, a, and p*** involve the extrinsic parameters *R,T*
  - For uncalibrated system:
    - A,a can be determined from Fund matrix (projective reconstr.)
    - **The vector *p* and the intrinsic parameters *K* are given by solving the above equation**

Outline

- Stratified auto calibration solves the 8 DOF in stages
  - Stage 1: solve for plane at infinity = ambiguity in R,T
  - Stage 2: solve for internal calibration matrix = IAC

Stage 1

solve for plane at infinity = ambiguity in R,T

- Modulus constraints:
  - Assume constant intrinsics in AC equations
  - Derive a necessary condition
  - Simultaneous quartic and cubic equations on 3 DOF of **p**
  - Local Minimization
- **Problem solved in this paper:**
  - Global Minimization of modulus constraints

Stage 2
solve for internal calibration matrix = IAC

- Solving the AC equations, given *p*

$$KK^\top = (A^i - a^i p^\top) KK^\top (A^i - a^i p^\top)^\top$$

- Previous work: Linear methods
  - Write the above equations as

    $$A\mathbf{x} = 0$$

    where x has the entries of $KK^\top$
  - Coerce the result x to be positive-definite, having form $KK^\top$
  - Sensitive to noise and to *p.*
- **Problem solved in this paper:**
  - Global minimization of $||A\mathbf{x}||$ subject to positive-definiteness

Experiments

- Synthetic data
  - 100 3D points within cube of side 20mm
  - Cameras ramdomly positioned at 40mm
  - Noise is added to coordinates
  - Randomly distort cameras within a synthetic ambiguity
    - Intrinsic calibration is identity
    - Remaining 3DOF (plane at infinity) is random

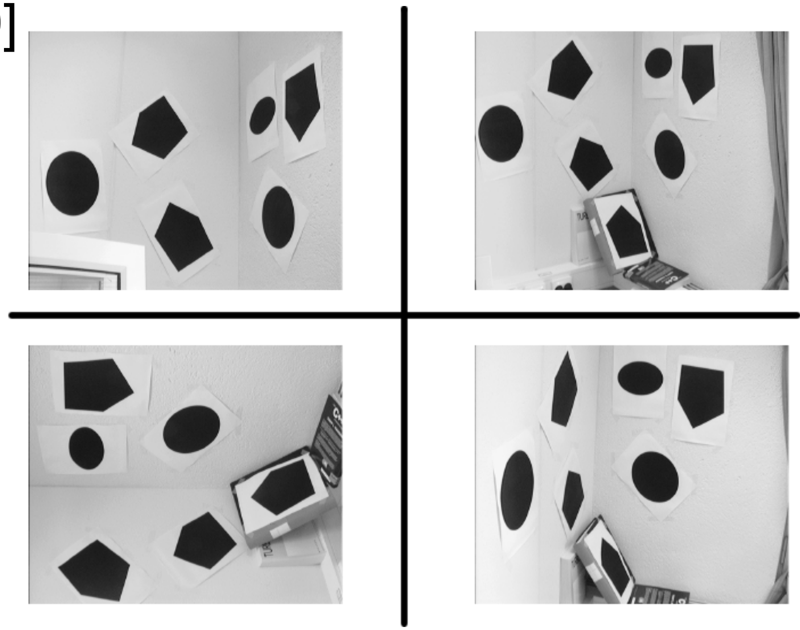
σ	m	Error				Iterations			Failure
(%)		Δp	Δf	Δuv	Δs	$\pi_\infty(1)$	$\pi_\infty(2)$	ω^*	(%)
0.1	5	3.13e-3	6.70e-3	4.34e-3	2.35e-3	22.5 ± 14.7	3.7 ± 4.4	37.5 ± 14.4	0
	10	8.45e-4	8.07e-4	1.02e-3	3.85e-4	14.5 ± 4.6	2.4 ± 2.4	40.9 ± 13.1	0
	20	8.39e-4	7.41e-4	3.90e-4	2.73e-4	14.4 ± 5.7	1.4 ± 1.2	31.8 ± 14.8	0
	40	7.65e-4	4.24e-4	2.82e-4	2.01e-4	13.8 ± 4.4	1.5 ± 3.2	27.5 ± 16.9	0
0.5	5	6.64e-3	7.50e-3	4.97e-3	2.33e-3	24.7 ± 15.3	4.5 ± 5.8	41.0 ± 16.2	0
	10	3.03e-3	2.51e-3	1.81e-3	1.20e-3	15.7 ± 5.6	3.0 ± 3.9	43.9 ± 15.9	0
	20	4.06e-3	2.13e-3	1.63e-3	1.10e-3	16.4 ± 10.5	4.1 ± 12.0	38.5 ± 11.6	0
	40	4.00e-3	1.69e-3	1.39e-3	7.09e-4	18.0 ± 9.0	9.3 ± 13.0	33.5 ± 11.7	0
1.0	5	1.09e-2	1.60e-2	7.03e-3	4.96e-3	25.0 ± 20.3	5.1 ± 10.4	42.7 ± 17.4	2
	10	5.81e-3	4.05e-3	3.01e-3	2.03e-3	18.6 ± 8.5	5.7 ± 8.8	44.3 ± 12.0	1
	20	8.16e-3	4.01e-3	2.44e-3	1.69e-3	21.5 ± 12.9	13.2 ± 17.2	43.6 ± 13.2	0
	40	7.80e-3	3.23e-3	2.37e-3	1.41e-3	24.3 ± 9.7	20.5 ± 13.7	43.3 ± 13.9	0

$$\Delta p = \sqrt{\sum_{i=1}^{3} (p_i/p_i^0 - 1)^2}, \quad \Delta f = \left| \frac{f_1 + f_2}{f_1^0 + f_2^0} - 1 \right|$$

$$\Delta uv = \big| (|u| + |v|) - (|u^0| + |v^0|) \big| / 2, \quad \Delta s = |s - s^0|$$

Experiments

- Time (matlab)
  - Typically 15s per iteration and 10 views, P4 1.5 GHz
- Real data
  - Bounds:
    - Focal length, aspect ratio [500,1500]
    - Principal point [185-250, 385-450]
    - Skew [-0.1 0.1]
  - Angles between walls
    - True: 88.1deg
    - Method: 89.8 deg



Extra Material

- Auto calibration equations
  - Cameras in projective reconstruction can be written as

$$\mathbf{P}_1 = [\mathbf{I}|\mathbf{0}] \qquad \mathbf{P}_i = [\mathbf{A}_i|\mathbf{a}_i]$$

$$\text{metric: } \mathbf{P}_i^M = \mathbf{K}[\mathbf{R}|\mathbf{t}] = \mathbf{P}_i\mathbf{H}$$

  - Ambiguity encoded in H as:

$$H = \begin{bmatrix} K & \mathbf{0} \\ -\mathbf{p}^\top K & 1 \end{bmatrix}$$

Noise (%)	m	Affine (Parallel)	Metric (Parallel)	Metric (Perpendicular)	Failure (%)
0.1	5	0.49 ± 0.13	0.49 ± 0.12	0.45 ± 0.33	1
	10	0.31 ± 0.07	0.31 ± 0.07	0.24 ± 0.06	0
	20	0.21 ± 0.04	0.21 ± 0.04	0.17 ± 0.03	0
0.5	5	2.32 ± 0.50	2.33 ± 0.52	2.02 ± 0.87	3
	10	1.50 ± 0.30	1.50 ± 0.30	1.14 ± 0.25	0
	20	1.07 ± 0.17	1.07 ± 0.17	0.81 ± 0.15	0
1.0	5	5.34 ± 1.57	5.36 ± 1.63	5.70 ± 4.00	10
	10	3.17 ± 0.63	3.18 ± 0.63	2.48 ± 0.66	0
	20	2.05 ± 0.39	2.05 ± 0.38	1.60 ± 0.38	0